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Classification of Lotka-Volterra cubic systems with 1:-2 singularity and straight lines of three directions and total multiplicity six

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We consider the real Lotka-Volterra cubic system of differential equations with 1:-2 singularity

$$\begin{aligned} \dot{x} &= x(a_{30}x^2 + a_{21}xy + a_{12}y^2 + a_{20}x + a_{11}y + 1) \equiv P(x, y), \\ \dot{y} &= y(b_{21}x^2 + b_{12}xy + b_{03}y^2 + b_{11}x + b_{02}y - 2) \equiv Q(x, y), \end{aligned} \qquad (1)$$

where $gcd(P, Q) = 1$. In this paper we give a classification of systems (1) with six invariant straight lines of three directions.

Theorem 1. *The system (1) has invariant straight lines of three directions of total algebraic multiplicity six if and only if it has one of the following twelve forms :*

1) $\dot{x} = -x(x - 1)$, $\dot{y} = y(y - 1)(2 - x - y)$;

2) $\dot{x} = x(1 - axy + (1 + d)y^2 + ax)$, $\dot{y} = y(y - 1)(dy + 2)$;

3) $\dot{x} = x(x - 1)(ax - 1)$, $\dot{y} = y(-2 + 3x + dy + (a - 2)x^2 - dxy)$;

4) $\dot{x} = \frac{1}{4}x(4 + f^2x^2 + 4fxy + 4(1 + c)y^2 - 4fx)$, $\dot{y} = y(y - 1)(2 + cy)$;

5) $\dot{x} = \frac{1}{4c^2}x(4f^2x^2 + 4c^2fxy + c^3(4 + c)y^2 + 8cfx + 4c^2)$,
$\dot{y} = y(y - 1)(cy + 2)$;

6) $\dot{x} = x(x - 1)(ax - 1)$,
$\dot{y} = -\frac{1}{8a^2}(16a^2 - 24a^3x - 8a^3(1 + 2a)x^2 + 8afy - 8a^2fxy + f^2y^2)$;

7) $\dot{x} = x(x - 1)(ax - 1)$,
$\dot{y} = -\frac{1}{8}y(f^2y^2 - 8fxy + 8(2 - a)x^2 - 24x + 8fy + 16)$;

8) $\dot{x} = \frac{1}{4}x(f^2x^2 + 4fxy - 4y^2 - 4fx + 4)$, $\dot{y} = -2y(y - 1)^2$;

9) $\dot{x} = x(x - 1)^2$, $\dot{y} = -\frac{1}{b^2}y(2b^2 - 3b^2x + b^2x^2 - 4by + 4bxy + 2y^2)$;

10) $\dot{x} = \frac{1}{2}x(x - 1)((2 + c)x - 2)$, $\dot{y} = \frac{1}{2}y(y - 1)(4 - (4 + c)x + 2cy)$;

11) $\dot{x} = \frac{1}{2+c}x(x - 1)(-2 + 2x - c)$,
$\dot{y} = \frac{1}{2+c}y(y - 1)(4 + 2c - (4 + c)x + 2cy + c^2y)$;

12) $\dot{x} = x(x - 1)(ax - 1)$,
$\dot{y} = -\frac{1}{9a}y(18a - 9a(1 + a)x + 6f(1 + a)y - 9afxy + 2f^2y^2)$;

where $abcdf(a - 1)(c + 2)(d + 2) \neq 0$ *and* $a, b, c, d, f \in \mathbb{R}$.