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On existence and uniqueness of mild solution to the initial-value problem for one neutral functional stochastic differential equation in $L_2^{\rho}(\mathbb{R}^d)$

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Many works are dedicated to investigation questions of existence and uniqueness of solution to SDEs under some given initial-boundary conditions in various functional spaces. There exists especial interest for ***neutral SDEs***. An essential feature of these equations is the phenomena of delay under so-called "derivative". In [1] its authors have considered an initial-value problem for an abstract SDE of such type in Hilbert space and have proved theorem on existence and uniqueness of its ***mild solution***. However, conditions of this theorem in a general form are rather difficult to check while solving specific applied problems. Therefore it is important to find conditions, convenient to check, which are expressed in terms of coefficients of the equation under investigation. It is possible to do it only in some particular cases, one of which is investigated.

For the initial-value problem for the following nonlinear neutral SDE

$$d\Bigg(u(t,x) + \int\limits_{\mathbb{R}^d} b(t,x,u(\alpha(t),\xi),\xi)d\xi\Bigg) = \big(\Delta_x u(t,x) + f(t,u(\alpha(t)),x)\big)dt +$$
$$+ \sigma(t,u(\alpha(t)),x)dW(t,x),\ 0 < t \le T,\ x \in \mathbb{R}^d,\ T > 0,$$
$$u(t,x) = \phi(t,x),\ \ -r \le t \le 0,\ x \in \mathbb{R}^d,\ r > 0,$$

where f, σ, b, ϕ, α are some real functions, and W – $L_2(\mathbb{R}^d)$-valued Wiener process, the theorem on existence and uniqueness of its ***mild solution*** in $\mathfrak{B}_{p,T,\rho}$, – Banach space of processes $\Phi\colon [0,T]\times\Omega \to L_2^{\rho}(\mathbb{R}^d)$ with the norm $\|\Phi\|_{\mathfrak{B}_{p,T,\rho}} = \sqrt[p]{\sup\limits_{0\le t\le T} \mathbf{E}\,\|\Phi(t)\|^p_{L_2^{\rho}(\mathbb{R}^d)}}$, $p \ge 2$, – is proved.

[1] Mahmudov N. I. *Existence, Uniqueness and Controllability Results for Neutral FSDEs in Hilbert Spaces* / A. M. Samoilenko, N. I. Mahmudov, A. N. Stanzhitskii // Dynamic Systems and Applications. – 2008. – **V. 17**. – P. 53 – 70.