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Optimal resource coefficient control in a dynamic population model without initial conditions

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Let $\Omega \subset \mathbb{R}^n$ $(n \in \mathbb{N})$ be a bounded domain with a regular boundary Γ, $S := (-\infty, 0]$, $Q := \Omega \times S$, $\Sigma := \Gamma \times S$. For arbitrary $\nu \in \mathbb{R}$, and $q \geq 1$, and a Hilbert space X, we denote by $L^q_\nu(S; X)$ the space of functions $f : S \to X$ such that $\int_S e^{-2\nu t} \|f(t)\|^q_X dt < \infty$.

Let $U := L^\infty(Q)$ be a space of controls and $U_\partial := \Big\{ v \in U \,\Big|\, 0 \leq v \leq M \quad \text{a.e. in} \quad Q \Big\}$ be the set of admissible controls. Given control $v \in U_\partial$ and $\lambda \in \mathbb{R}$, the state $y = y(v) \in L^2_\lambda(S; H^1_0(\Omega)) \cap L^p_\lambda(S; L^p(\Omega)) \cap C(S; L^2(\Omega))$ of controlled evolution system is defined as a weak solution of the problem

$$y_t - \sum_{i,j=1}^{n} (a_{ij}(x) y_{x_i})_{x_j} + c(x)|y|^{p-2} y - v(x,t) y = f(x,t), \quad (x,t) \in Q,$$

$$y\big|_\Sigma = 0, \qquad \lim_{t \to -\infty} e^{-2\lambda} \int_\Omega |y(x,t;v)|^2 \, dx = 0.$$

The cost function has the form

$$J(v) = \iint_Q \big[\, |y(x,t;v)| - \rho(x,t)|v|^2 \,\big] \, dxdt, \quad v \in U,$$

where $\rho \in L^1(Q)$ is given.

The **problem** is to find element $u \in U_\partial$ such that

$$J(u) = \sup_{v \in U_\partial} J(v) \, .$$

Under the additional conditions, we prove the existence of a solution of this problem.