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Maximal multiplicity of the line at infinity for cubic differential systems with two real non-parallel invariant straight lines

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We consider the real cubic system of differential equations

$$\begin{array}{c} \dot{x} = \sum_{j=0}^{3} P_j(x,y) \equiv P(x,y),\ \dot{y} = \sum_{j=0}^{3} Q_j(x,y) \equiv Q(x,y), \\ \gcd(P,Q)=1,\ yP_3(x,y) - xQ_3(x,y) \not\equiv 0. \end{array} \quad (1)$$

The straight line $l \equiv \alpha x + \beta y + \gamma = 0,\ \alpha,\beta,\gamma \in \mathbb{C},\ (\alpha,\beta) \neq (0,0)$ is called invariant for (1) if there exists a polynomial $K(x,y)$ such that the identity $\alpha P + \beta Q \equiv l \cdot K(x,y)$ holds. We say that invariant straight line l (the line at infinity $z=0$) has multiplicity $m \in \mathbb{N}^*$ if m is the greatest number such that l^m (z^{m-1}) divides $R(x,y) = P^2Q'_x - Q^2P'_y + PQ(Q'_y - P'_x)$ (the homogenized polynomial $z^8R(x/z,y/z)$).

1 *The maximal multiplicity of the line at infinity for cubic system* (1) *with two real non-parallel invariant straight lines is at most three. Any such system for which the line at infinity has multiplicity three via an affine transformation and time rescaling can be written in one of the following four forms:*

$$\begin{array}{c} \dot{x} = x, \quad \dot{y} = y(a+bx+cy+dx^2+exy+fy^2), \\ (a^2+c^2+f^2)(d^2+e^2+f^2)(a^2+b^2+d^2)((a-1)^2+c^2+f^2)\cdot \\ ((a-1)^2+b^2+d^2)((a-1)^2+(c^2d-bce+b^2f)^2) \neq 0; \end{array} \quad (2)$$

$$\begin{array}{c} \dot{x} = x(a+by+cxy+y^2),\ \dot{y} = -y(d+ex+c^2x^2+cxy), \\ ad(c^2+e^2+(a+d)^2)((a+d)^2+(bc-e)^2) \neq 0; \end{array} \quad (3)$$

$$\begin{array}{c} \dot{x} = x(a+by+cxy+dy^2),\ \dot{y} = \alpha y(1+bx+cx^2+dxy), \\ \alpha a(c^2+d^2)(\alpha - a) \neq 0; \end{array} \quad (4)$$

$$\begin{array}{c} \dot{x} = x(a+by), \quad \dot{y} = y(c+dx+ey+x^2), \\ a(c^2+e^2)((a-c)^2+(b-e)^2) \neq 0. \end{array} \quad (5)$$

For the systems (2)-(5) the lines $x=0$ and $y=0$ are invariant straight lines of the multiplicity one.

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