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# The stability of the fixed points of discrete dynamical systems in space conv $\mathbb{R}^n$

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Let conv $\mathbb{R}^n$ be a metric space of convex compact sets from $\mathbb{R}^n$, in which the Hausdorff metric $d_H$ is introduced. Consider the discrete dynamical system (DDS) in conv $\mathbb{R}^n$

$$\overline{u} = G(u, V[u]), \quad u \in \text{conv } \mathbb{R}^n, \quad G : \text{conv } \mathbb{R}^n \times \mathbb{R}_+ \to \text{conv } \mathbb{R}^n, \quad (1)$$

where $V[u]$ is a volume of compact $u$, the map $G$ satisfies the following condition: for any $(u_0, v_0) \in \text{conv } \mathbb{R}^n \times \mathbb{R}_+$ there exists a $G'_u(u_0, v_0) \in L(C(S^{n-1}))$, $G'_V(u_0, v_0) \in C(S^{n-1})$ and a neighbourhood $\mathcal{U} \subset \text{conv } \mathbb{R}^n \times \mathbb{R}$ such that, for all $(u, v) \in \mathcal{U}$ we have

$$G(u, v) = G(u_0, v_0) + G'_u(u_0, v_0)\Delta u + G'_v(u_0, v_0)\Delta v + o(\|(\Delta u, \Delta v)\|),$$

where $\Delta u = u - u_0$, $\Delta v = v - v_0$, $\|(u, v)\| = \|u\|_{C(S^{n-1})} + |v|$, $L(C(S^{n-1}))$ is a Banach algebra of bounded linear operators on a Banach space $C(S^{n-1})$.

Suppose that there is a fixed point $u^* \in \text{conv } \mathbb{R}^n$ of DDS (1), that is $u^* = G(u^*, V[u^*])$.

Let $D(\lambda) = 1 - \int\limits_{S^{n-1}} R(\lambda, G'_u) G'_V F[u^*; d\omega]$, $\lambda \in \rho(G'_u)$, where $R(\lambda, .)$ is a resolvent, $\rho(.)$ is a resolvent set and $\sigma(.)$ is a spectrum of the corresponding operator, $G'_u = G'_u(u^*, v^*)$, $G'_V = G'_V(u^*, v^*)$, $v^* = V[u^*]$, $F[u^*, d\omega]$ is a surface function $u^*$ [1].

**Theorem 1** [2] *Assume that* $\sigma(G'_u) \cup \{\lambda \in \rho(G'_u) \mid D(\lambda) = 0\} \subset \{\lambda \in \mathbb{C} \,|\, |\lambda| < 1\}$. *Then the fixed point* $u^*$ *of the DDS (1) is asymptotically stable.*

The stability conditions are concretized for DDS in the space conv $\mathbb{R}^2$. In particular, the cases when the analysis of the stability of a fixed point is reduced to the study of roots location of the characteristic polynomial are established.

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