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Center conditions for non-linear differential systems

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We consider the system of differential equations

$$\frac{dx}{dt} = -y, \qquad \frac{dy}{dt} = x + xf(x) + g(x,y) \equiv x + Q(x,y), \tag{1}$$

where $g(x,y) = y\hat{g}(x,y)$, $\hat{g}(0,0) = f(0) = 0$, and $Q(x,y)$ is analytical function in a neighborhood of the origin $O(0,0)$ of system of coordinates, i.e. $Q \in C^{\omega}(0,0)$. The origin $(0,0)$ is a singular point for (1) of a center or a focus type. The problem of distinguishing these types arise.

In this work we give necessary and sufficiency conditions such that $(0,0)$ to be a center type for (1).

We write the system (1) in the form

$$\frac{dx}{dt} = -y, \quad \frac{dy}{dt} = \frac{x}{\varphi}(1 - \mathbb{B}), \tag{2}$$

where $\varphi = 1/(1+f)$, $\mathbb{B} = -\varphi g/x = \sum\limits_{i=1}^{\infty} B_i(x)y^i$. We denote

$$\begin{aligned}
&\delta(\psi) = \varphi\psi'_x/x, \ \ \beta_1 = B_1, \ \ \alpha_2 = B_1^2 + \tilde{f}_0, \ \ \ldots \ \ , \alpha_{2i} = \\
&2(\tilde{f}_0 - B_2)\alpha_{2(i-1)} + (2^{i-1}i!)(B_{2i} + \sum_{k=1}^{i-1}\alpha_{2k}B_{2(i-k)}/(2^{k-1}k!) + \\
&\sum_{k=0}^{i-1}\beta_{2k+1}B_{2(i-k)-1}/(2k+1)!!) + \delta(\alpha_{2(i-1)}), \\
&\beta_{2i+1} = (2i+1)!!(B_{2i+1} + \sum_{k=1}^{i}\alpha_{2k}B_{2(i-k)+1}/(2^{k-1}k!) + \\
&\sum_{k=0}^{i-1}\beta_{2k+1}B_{2(i-k)}/(2k+1)!!) + 2(\tilde{f}_0 - B_2)\beta_{2i-1} + \delta(\beta_{2i-1}), \ldots
\end{aligned} \tag{3}$$

Theorem. *The singular point $O(0,0)$ is a center type for differential system (2) if and only if there is a function $\tilde{f}_0 \in C^{\omega}(0)$ such that the functions α_{2i}, β_{2i+1}, defined by relations (3), are analytical in $x = 0$.*

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