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Investigation of non-linear boundary value problems by parametrization at multiple nodes

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Here, we study the absolute continuous solution of non-linear boundary value problem

$$\frac{du(t)}{dt} = f\left(t, u(t)\right), \ t \in [a, b], \quad \Phi(u) = d, \tag{1}$$

where $f : [a, b] \times D \to \mathbb{R}^n, D \subset \mathbb{R}^n$ is a Caratheodory function, $d \in \mathbb{R}^n$ is a given vector, and Φ is a vector functional on the space of absolutely continuous functions (generally speaking, non-linear).

Following the idea used in numerical methods for approximate solution of initial value problem for ordinary differential equations, let us choose $N+1$ grid points $t_0 = a, t_k = t_{k-1}+h_k, k = 1, 2, ..., N-1, t_N = b$, where $h_k, \ k = 1, 2, ..., N-1$, are the corresponding step sizes.

The idea that we are going to use suggests to replace the original non-local problem (1) on each interval $[t_{k-1}, t_k], k = 1, 2, ..., N$ by a suitable family of two initial value problems:

$$\begin{array}{l} \frac{dx^{(k)}}{dt} = f\left(t, x^{(k)}\right), \ t \in [t_{k-1}, t_k], \\ x(t_{k-1}) = z^{(k-1)}, \ x\left(t_k\right) = z^{(k)}, k = 1, 2, ..., N, \end{array} \tag{2}$$

where vectors $z^{(k)} = col(z_1^{(k)}, z_2^{(k)}, ..., z_n^{(k)}), \ k = 0, 1, 2, ..., N$ will be considered as unknown parameters whose values are to be determined.

Due to the form of the transformed additional conditions, it is natural to apply to (2) the successive approximation techniques similar to those used in [1], [2]. The specific properties of restrictions Φ being transferred to so called determining algebraic equations.

[1] Ronto A., Ronto M. and Shchobak N., *Notes on interval halving procedure for periodic and two-point problems*, Boundary Value Problems, (2014), pp. 20.

[2] Ronto A., Ronto M. and Varha J., *A new approach to non-local boundary value problems for ordinary differential systems*, Applied Mathematics and Computation, (2015), pp. 689-700.