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Exponential dichotomy and singularly perturbed operator differential equation in the Hilbert space

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The report is devoted to obtaining the conditions of the existence of solutions of boundary value problem

$$\varepsilon\frac{dx(t,\varepsilon)}{dt} = Ax(t,\varepsilon) + \varepsilon B(t)x(t,\varepsilon) + \Phi(t), t \in J, \tag{1}$$

$$lx(\cdot,\varepsilon) = \alpha, \tag{2}$$

and boundary value problem

$$\frac{dx(t,\varepsilon)}{dt} = C(t)x(t,\varepsilon) + \varepsilon B(t)x(t,\varepsilon) + \Phi(t), \tag{3}$$

$$lx(\cdot,\varepsilon) = \alpha. \tag{4}$$

Here J is a subset of $R, J \subset R$, $x(t,\varepsilon)$ is an unknown solution from the space $BC^1(J,H)$ (the space of bounded and continuous functions together with its derivative). $B(t), C(t)$ are smooth operator valued functions, $\Phi(t)$ is a smooth vector function, $A \in \mathcal{L}(H)$;

$$l : BC^1(J,H) \to H_1$$

is a linear and bounded operator from the space $BC^1(J,H)$ into the Hilbert space H_1; $\alpha \in H_1$. We seek the solution of (1), (2) in the form of a series [1], [2]

$$x(t,\varepsilon) = \sum_{i=0}^{+\infty} \varepsilon^i x_i(t),$$

which turns (for $\varepsilon = 0$) in one of the solutions of generating operator boundary value problem

$$Ax_0(t) + \Phi(t) = 0,$$
$$lx_0(\cdot) = \alpha.$$

Boundary value problem (3), (4) is investigated under assumption that the generating homogeneous equation ($\varepsilon = 0$)

$$\frac{dx_0(t)}{dt} = C(t)x_0(t)$$

admits an exponential dichotomy on the semi-axes $\mathbb{R}_+$, $\mathbb{R}_-$ [3] and solution $x(t, \varepsilon)$ turns in one of solutions of boundary value problem

$$\frac{dx(t)}{dt} = C(t)x(t) + \Phi(t),$$
$$lx(\cdot) = \alpha.$$

[1] Boichuk A.A., Samoilenko A.M. *Generalized invertible operators and Fredholm Boundary-Value Problems*, VSP, Utrecht-Boston, 2004.

[2] Boichuk A.A., Pokutnyi O.O. *Bounded solutions of linear perturbed differential equations in a Banach space*, Tatra Mt. Math. Publ. 38 (2007), 29-40.

[3] Pokutnyi A.A. *Bounded solutions of linear and weakly nonlinear differential equations in a Banach space with unbounded operator in the linear part*, Differential equations, vol.48, No6, 2012, pp.803-813.