Zinovii Nytrebych [1], Oksana Malanchuk [2]

Homogeneous Two-point in Time Problem for PDE

[1] *Lviv Polytechnic National University, Lviv, Ukraine*
E-mail: znytrebych@gmail.com
[2] *Danylo Halytsky Lviv National Medical University, Lviv, Ukraine*
E-mail: Oksana.Malan@gmail.com

In the domain $(t, x_1, \ldots, x_s) \in \mathbb{R}^{1+s}$, $s \in \mathbb{N}\setminus\{1\}$, we investigate the set of solutions $U = U(t,x)$ of the following PDE

$$L\Big(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}\Big)U \equiv \frac{\partial^2 U}{\partial t^2} + 2a\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t} + b\Big(\frac{\partial}{\partial x}\Big)U = 0, \tag{1}$$

in which operator coefficients $a\left(\frac{\partial}{\partial x}\right)$, $b\left(\frac{\partial}{\partial x}\right)$ are arbitrary differential expressions with complex coefficients of finite or infinite order, their symbols $a(\nu)$, $b(\nu)$ are entire functions of complex vector-variable $\nu \in \mathbb{C}^s$.

Among the solutions of equation (1) we find the ones solutions that satisfy the following homogeneous local two-point conditions:

$$\begin{aligned} &A_1\Big(\frac{\partial}{\partial x}\Big)U(0,x) + A_2\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t}(0,x) = 0, \\ &B_1\Big(\frac{\partial}{\partial x}\Big)U(h,x) + B_2\Big(\frac{\partial}{\partial x}\Big)\frac{\partial U}{\partial t}(h,x) = 0, \qquad h > 0, \quad x \in \mathbb{R}^s, \end{aligned} \tag{2}$$

where $A_1\left(\frac{\partial}{\partial x}\right)$, $A_2\left(\frac{\partial}{\partial x}\right)$, $B_1\left(\frac{\partial}{\partial x}\right)$, $B_2\left(\frac{\partial}{\partial x}\right)$ are differential polynomials with complex coefficients, whose symbols $A_1(\nu)$, $A_2(\nu)$, $B_1(\nu)$, $B_2(\nu)$ for $\nu \in \mathbb{C}^s$ satisfy conditions: $|A_1(\nu)|^2 + |A_2(\nu)|^2 \neq 0$, $|B_1(\nu)|^2 + |B_2(\nu)|^2 \neq 0$.

We prove that problem (1), (2) has only trivial solution in the case when the characteristic determinant of the problem is nonzero. In another case, we prove the existence of nontrivial solutions of the problem and propose the differential-symbol method [1] of constructing them.

[1] P. I. Kalenyuk, Z. M. Nytrebych, *Generalized scheme of separation of variables. Differential-symbol method*, Lviv Polytechnic, Lviv, 2002 (in Ukrainian).