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Centro-affine invariants and stability of unperturbed motion in ternary polynomial differential systems

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We consider the ternary differential system

$$\frac{dx^j}{dt} = a^j_\alpha\, x^\alpha + \sum_{i=1}^{l} a^j_{\alpha_1\alpha_2\ldots\alpha_{m_i}}\, x^{\alpha_1} x^{\alpha_2} \ldots x^{\alpha_{m_i}} \qquad (j,\alpha,\alpha_1,\alpha_2,\ldots,\alpha_{m_i} = 1,2,3;\ l<\infty), \tag{1}$$

where $a^j_{\alpha_1\alpha_2\ldots\alpha_{m_i}}$ is a symmetric tensor in lower indices in which the total convolution is done and $\{m_1, m_2, \ldots, m_l\}$ $(m_i \geq 2)$ is a finite set of distinct natural numbers. In [1] it was proved that

$$\theta_1 = a^\alpha_\alpha, \quad \theta_2 = a^\alpha_\beta a^\beta_\alpha, \quad \theta_3 = a^\alpha_\gamma a^\beta_\alpha a^\gamma_\beta$$

are centro-affine invariants of (1) with respect to the group $GL(3,\mathbb{R})$.

The characteristic equation of system (1) and of the first approximation system $\frac{dx^j}{dt} = a^j_\alpha\, x^\alpha$ $(j,\alpha = 1,2,3)$ [2] can be written as

$$\rho^3 + L_{1,3}\,\rho^2 + L_{2,3}\,\rho + L_{3,3} = 0, \tag{2}$$

where

$$L_{1,3} = -\theta_1,\ L_{2,3} = -\frac{1}{2}\,(\theta_2 - \theta_1^2),\ L_{3,3} = -\frac{1}{6}\,(\theta_1^3 - 3\theta_1\theta_2 + 2\theta_3). \tag{3}$$

Using the Lyapunov theorems on stability of motion in the first approximation as well the Hurwitz theorem to the root analysis of the characteristic equation (2) (see for example [2]) there were proved the following theorems

1 *If the centro-affine invariants of (1) satisfy the following conditions*

$$L_{1,3} > 0, \quad L_{2,3} > 0, \quad L_{1,3}L_{2,3} - L_{3,3} > 0,$$

then irrespective of terms of order higher than one, the unperturbed motion is asymptotically stable.

2 *If at least one of the centro-affine invariant expression (3) of system (1) is negative, then irrespective of terms of order higher than one, the unperturbed motion is unstable.*

Following [3] for Lyapunov-Darboux system with quadratic nonlinearities [4], the centro-affine invariant conditions for the existence of periodic unperturbed solutions were obtained. Moreover, it was established that every other motion near the unperturbed solution will tend asymptotically to one of the periodic motions.

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