Halyna Melnyk

Evolutionary game analysis of demand dynamic in oligopolistic competition

Yuriy Fedkovych Chernivtsi National University, Chernivtsi, Ukraine
E-mail: firstmy555@gmail.com

The non-cooperative dynamic oligopolistic competition amongst the service providing agents may be placed in the form of a dynamic variational inequality (DVI) [1]. We denote the set of revenue managing firms as F, each of whom is providing a set of services S. Continuous time is denoted by the scalar $t \in \Re^1_+$, while t_0 is the finite initial time and $t_1 \in \Re^1_{++}$ the finite terminal time so that $t \in [t_0, t_1] \subset \Re^1_+$. Each firm $f \in F$ controls price $\pi_s^f \in L^2[t_o, t_1]$ corresponding to each service type $s \in S$, where $L^2[t_o, t_1]$ is the space of square-integrable functions for the real time interval $t \in [t_0, t_1] \subset \Re^1_+$. The demand for the service offerings of firm $f \in F$ evolve according to the following evolutionary game-theoretic dynamics [2]:

$$\frac{dN_s^f}{dt} = \eta_s^f \cdot \left(\pi_s - \pi_s^f\right) \;\; \forall s \in S,\, f \in F \tag{1}$$

$$N_s^f(t_0) = K_{s,0}^f \;\; \forall s \in S,\, f \in F \tag{2}$$

where π_s is the moving average price for service $s \in S$ given by

$$\pi_s(t) = \frac{1}{|F|\,(t - t_0)} \int_{t_0}^{t} \sum_{g \in F} \pi_s^g(\tau) d\tau \;\; \forall s \in S$$

while $K_{i,0}^f \in \Re^1_{++}$ and $\eta_i^f \in \Re^1_{++}$ are exogenous parameters for each $s \in S$ and $f \in F$.

The resulting Cournot-Nash game takes the form of a differential variational inequality and can be solved by a numerical method.

[1] T. L. Friesz and R. Mookherjee, *Differential variational inequalities with state-dependent time shifts*, Tech. Rep., Dept. of Industrial and Manufacturing Eng., Pennsylvania State University, 2004.

[2] D. Fudenberg and D. K. Levine, *The Theory of Learning in Games, Second edition*, The MIT Press, 1999.