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Initial-boundary value problems for nonlinear parabolic equations with time depended delay

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Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with a piecewise smooth boundary $\partial\Omega$, $\partial\Omega = \Gamma_0 \cup \Gamma_1$, $\Gamma_1 \cap \Gamma_0 = \varnothing$, $\nu = (\nu_1, \dots, \nu_n)$ is a unit outward pointing normal vector on the $\partial\Omega$; $T > 0$ and $Q := \Omega \times (0,T)$, $\Sigma_0 := \Gamma_0 \times (0,T)$, $\Sigma_1 := \Gamma_1 \times (0,T)$.

Consider the problem of finding a function $u : \overline{\Omega} \times [-\tau_0, T] \to \mathbb{R}$ satisfying (in some sense) the equation

$$u_t - \sum_{i=1}^{n} \frac{d}{dx_i} a_i(x,t,u,\nabla u) + a_0(x,t,u,\nabla u) + \int\limits_{t-\tau(t)}^{t} c(x,t,s,u(x,s))ds$$

$$= -\sum_{i=1}^{n} \frac{\partial}{\partial x_i} f_i(x,t) + f_0(x,t), \quad (x,t) \in Q, \tag{1}$$

the boundary conditions

$$u\Big|_{\Sigma_0} = 0, \qquad \sum_{i=1}^{n} a_i(x,t,u,\nabla u)\,\nu_i\Big|_{\Sigma_1} = 0, \tag{2}$$

and the initial condition

$$u(x,t) = u_0(x,t), \quad (x,t) \in \overline{\Omega} \times [-\tau_0, 0]. \tag{3}$$

Here $\tau : [0,T] \to \mathbb{R}$ is a continuous function such that $\tau(t) \geq 0$ for all $t \in [0,T]$, $\tau_0 := \max\{-\inf\limits_{t\in[0,T]}(t - \tau(t)), 0\}$ (assume $[-0,0] = \{0\}$), and $a_i : Q \times \mathbb{R}^{1+n} \to \mathbb{R}$, $c : Q \times (-\tau_0, T) \times \mathbb{R} \to \mathbb{R}$, $f_i : Q \to \mathbb{R}$ $(i = \overline{0,n})$, $u_0 : \overline{\Omega} \times [-\tau_0, 0] \to \mathbb{R}$ are given real-valued functions from the corresponding classes of initial data.

The existence and uniqueness of a weak solutions of the problem are proved and its a priori estimate is obtained.