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The convolution of Laurent formal series and a fundamental solution of an implicit linear differential equation over an arbitrary ring

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In the work we consider the convolution operation in the ring $K((\frac{1}{x}))$ of Laurent formal series having the form

$$b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0 + \frac{a_1}{x} + \frac{a_2}{x^2} + \ldots + \frac{a_n}{x^n} + \ldots$$

over an arbitrary ring K, which is not necessarily commutative, with an identity element. This operation is an algebraic analogue for the Hurwitz product of series, which is widely used in the theory of functions (see [1]). Using this convolution we obtain some formula of Cauchy type for the solution of the following implicit linear inhomogeneous differential equation

$$by' + R(x) = y, \tag{1}$$

where $b \in K$, with the inhomogeneity $R(x)$ in the form of a Laurent polynomial. This formula allows us to consider the Euler series

$$\mathcal{E}_b(x) = \frac{1}{x} - \frac{1!b}{x^2} + \frac{2!b^2}{x^3} - \frac{3!b^3}{x^4} \ldots$$

as a fundamental solution of the differential equation (1) in $K((\frac{1}{x}))$ and in $K[x, \frac{1}{x}]$. In the case K is an algebraically closed field of characteristic zero this formula is generalized to the case of arbitrary rational inhomogeneity.

We also consider the equation $by' + f(x) = y$, where $b \in \mathbb{Z}, f \in \mathbb{Z}[[x]]$, and show that the Euler series $\mathcal{E}_b(x)$ is its fundamental solution as well. To this end, we define the convolution of an element from $\frac{1}{x}\mathbb{Z}[[x]]$ with an element from $\mathbb{Z}[[x]]$, using the p-adic topology on $\mathbb{Z}$.

[1] L. Bieberbach. *Analitische Fortsetzung*, Springer – Verlag, 1955.