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Existence of the smallest eigenvalue of the eigenvalue problem for the Laplace-Beltrami operator on the unit sphere

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Let K be an open cone with vertex in the point $\mathcal{O}$ and the aperture $\omega_0 \in (0, \pi)$. We define $\Omega := K \cap S^{n-1}$. Let $\vec{\nu}$ be the unite exterior normal vector to ∂K at the points of $\partial\Omega$. Let $\chi(\omega) \geq 0$, $\gamma(\omega) > 0$ be $C^0(\partial\Omega)$ - functions.

In our presentation we will formulate and prove a theorem stating the existence of the smallest positive eigenvalue λ of the following eigenvalue problem for the Laplace - Beltrami operator Δ_ω on the unit sphere

$$\begin{cases} \Delta_\omega \psi + \lambda(\lambda + n - 2)\psi(\omega) = 0, & \omega \in \Omega \\ \frac{\partial \psi}{\partial \vec{\nu}} + \langle \lambda \chi(\omega) + \gamma(\omega) \rangle \, \psi(\omega) = 0, & \omega \in \partial\Omega \end{cases} . \qquad (EVP)$$

In addition we will show that the smallest eigenvalue of (EVP) satisfies the following inequality

$$1 < \lambda < \sqrt{\left(\frac{\pi}{\omega_0}\right)^2 + \left(\frac{n-2}{2}\right)^2} - \frac{n-2}{2}$$

for $n \geq 2$.

The ideas of the proof of this theorem are based on the Legendre spherical harmonics and the Gegenbauer functions.