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Qualitative method in the investigation of nonlinear oscillations of elastic beam

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We investigate the solution of the problem on nonlinear transverse vibrations of elastic body subject to the action of dissipative forces in a bounded domain. Such problems have many applications in various technical systems, such as vibration of pipelines, railway lines, drill columns, bridges, electrical wires, optical fiber etc. For the nonlinear oscillations model considered, there is no general analytical methodologies of determining dynamical characteristics of the oscillation process. Thus we propose using the qualitative methods of nonlinear boundary value problems theory to obtain the well-posedness conditions for the problem's local solution in time variable. The methodology of the qualitative study of nonlinear oscillations under the action of dissipative forces is based on the general principles of the nonlinear boundary value problems theory, such as monotony method and Galerkin method.

Let $Q_T = (0, l) \times (0, T)$, where $T < +\infty$, $l < +\infty$, $\tau \in (0, T)$. In the domain Q_T, for nonlinear equation

$$u_{tt} + au_{txxxx} + bu_{xxxx} + c|u_t|^{p-2}u_t = f(x, t), \quad p > 2 \tag{1}$$

we shall consider the mixed problem with initial conditions $u(x, 0) = u_0(x), \quad u_t(x, 0) = u_1(x)$ and boundary conditions $u(0, t) = u_{xx}(0, t) = 0, \quad u(l, t) = u_{xx}(l, t) = 0$. In equation (1), function $u(x, t)$ is a cross motion of the beam section with x coordinate at arbitrary time moment t; $a > 0$, $b > 0$ are constants which can be expressed through geometrical and physico-mechanical parameters of the beam, constant $c > 0$ describes the nonlinear elastic forces acting in the system, $f(x, t)$ is the external driving force. Boundary conditions correspond to the model of a beam with fixed pivoting supports on the ends $x = 0$ and $x = l$.

We investigate the oscillating system modeled by problem and obtain the conditions of existence of a local solution. We show the possibility of applying the Galerkin method to solving the problem.