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Asymptotic representations of solutions with slowly varying derivatives of essentially nonlinear differential equations of the n-th order

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The differential equation

$$y^{(n)} = \alpha_0 p(t) \prod_{i=0}^{n-1} \varphi_i(y^{(i)}) \exp\left(R\left(\ln(|y^{(n-1)}|\right)\right), \tag{1}$$

where $\alpha_0 \in \{-1, 1\}$, $p : [a, \omega[\to]0, +\infty[$ $(-\infty < a < \omega \leq +\infty)$, $\varphi_i : \Delta_{Y_i} \to]0, +\infty[$ $(i = 0, ..., n)$, are continuous functions, $R :]0, +\infty[\to]0, +\infty[$ is continuously differentiable function, $Y_i \in \{0, \pm\infty\}$, Δ_{Y_i} is either the interval $[y_i^0, Y_i[$, or the interval $]Y_i, y_i^0]$, is considered. If $\omega > 0$ we will take $a > 0$. We suppose also, that R is regularly varying function of index μ, every $\varphi_i(z)$ is regularly varying as $z \to Y_i$ $(z \in \Delta_{Y_i})$ of index σ_i and $0 < \mu < 1$, $\sigma_0 + ... + \sigma_n \neq 1$. If $Y_i = +\infty (Y_i = -\infty)$ we take $y_i^0 > 0$ $(y_i^0 < 0)$ correspondingly.

We call the solution y of the equation (1) the $P_\omega(Y_0, Y_1, ..., Y_{n-1}, \lambda_0)$-solution, where $-\infty \leq \lambda_{n-1}^0 \leq +\infty$, if the next conditions take place

$$y^{(i)} : [t_0, \omega[\to \Delta_{Y_i}, \quad \lim_{t\uparrow\omega} y^{(i)}(t) = Y_i \quad (i = 0, ..., n-1),$$

$$\lim_{t\uparrow\omega} \frac{(y^{(n-1)}(t))^2}{y^{(n)}(t)\, y^{(n-2)}(t)} = \lambda_{n-1}^0.$$

The cases $\lambda_0 \in \{0, \pm\infty\}$ are singular in studying of $P_\omega(Y_0, Y_1, ..., Y_{n-1}, \lambda_0)$-solutions of (1). Such solutions were investigated [1] only by some additional conditions to the right side of equation (1). The necessary and sufficient conditions of the existence and asymptotic representations as $t \uparrow \omega$ of $P_\omega(Y_0, Y_1, ..., Y_{n-1}, \infty)$-solutions of the equation (1) was established in general case.

[1] V.M. Evtukhov, A.M. Klopot,Asymptotic Representations for Some Classes of Solutions of Ordinary Differential Equations of Order n with Regularly Varying Nonlinearities *Ukrainian Mathematical Journal*, **65(3)**, (2013), pp. 354-380.